OC3140
HW/Lab 4 Probability

1. Select two basketball teams (of 5) from 15 students. How many ways can this be accomplished? If, there are two who will not play together on the same team, how many possible selections are available?

Solution:

(a). Method A:

The ways of selecting 10 basket players from 15 students are: C_{10}^{15}

The ways of selecting one team (5 players) from 10 players are: C_5^{10}

The total ways of selecting two basketball teams (of 5) from 15 students will be:

$$C_{10}^{15} \cdot C_5^{10} = 3003 \times 252 = 756756 .$$

Method B:

The ways of selecting 5 plays (one team) from 15 students are: C_5^{15}

The ways of selecting 5 plays for another team from the left 10 students are: C_5^{10}

The total ways of selecting two basketball teams (of 5) from 15 students will be:

$$C_5^{15} \cdot C_5^{10} = 3003 \times 252 = 756756$$

(b) . Method A:

The ways of selecting 2 students in team A are:

$$C_3^{13} \cdot C_5^{10} = \frac{13 \times 12 \times 11}{3 \cdot 2 \cdot 1} \times 252 = 72072 .$$

The ways of selecting 2 students in team B also be:

$$C_3^{13} \cdot C_5^{10} = \frac{13 \times 12 \times 11}{3 \cdot 2 \cdot 1} \times 252 = 72072 .$$

So the possible selections of two students will not play together on the same team will be:

$$C_{10}^{15} C_5^{10} - 2 C_3^{13} C_5^{10} = C_5^{10} \left(C_{10}^{15} - 2 C_3^{13} \right) = 612612 .$$

Method B:

Let us say, student Mike and Joe can not play in same team.

The ways of selecting with both Mike and Joe not play are: $C_5^{13} \cdot C_5^{8}$

The ways of selecting with Mike plays team A and Joe not play are: $C_4^{13} \cdot C_5^9$. The ways of selecting only one of those two students can be 4 cases: Mike plays team A, Mike plays team B, Joe plays team A, and Joe plays team B. So the total ways of only one of those two students plays basketball will be: $4 \cdot C_4^{13} \cdot C_5^9$.

The ways of Mike plays team A and Joe plays team B are: $C_4^{13} \cdot C_4^9$

The ways of Mike plays team B and Joe plays team A are: $C_4^{13} \cdot C_4^9$

The total ways of two students will not play in same team will be:

$$C_5^{13} \cdot C_5^8 + 4 \cdot C_4^{13} \cdot C_5^9 + 2 \cdot C_4^{13} \cdot C_4^9 = 612612.$$

2. The probability that an American industry will locate in San Jose is 0.7, the probability that it will locate in Monterey is 0.4, and the probability that it will locate in either San Jose or Monterey or both is 0.8. What is the probability that the industry will locate

   (a). in both cities?
   (b). in neither city?

Solution::

Definition:

S - will locate in San Jose
M - will locate in Monterey
$\bar{S}$ - will not locate in San Jose
$\bar{M}$ - will not locate in Monterey
$P(S)$ - the probability locate in San Jose (0.7)
$P(M)$ - the probability locate in Monterey (0.4)
$P(S \cup M)$ - the probability locate in either San Jose or Monterey (0.8)
$P(S \cap M)$ - the probability locate in both San Jose and Monterey
$P(\bar{S} \cap \bar{M})$ - the probability locate in neither San Jose nor Monterey

(a). in both cities:

$$P(S) = P(S \cap \bar{M}) + P(S \cap M)$$

$$P(M) = P(\bar{S} \cap M) + P(S \cap M)$$

$$P(S \cup M) = P(\bar{S} \cap M) + P(S \cap \bar{M}) + P(S \cap M)$$

$$P(S \cap M) = P(S) + P(M) - P(S \cup M) = 0.7 + 0.4 - 0.8 = 0.3$$

(b). in neither city:

$$P(\bar{S} \cap \bar{M}) = 1 - P(S \cup M) = 1 - 0.8 = 0.2$$

3. Consider a probability density function as: $f(x) = \begin{cases} k\sqrt{x} & 0 < x < 1 \\ 0 & elsewhere \end{cases}$

   (a) Evaluate. k
   (b) Find $F(x)$ and use it to evaluate $P(0.3 < x < 0.6)$.

Solution:

(a). $\int_{-\infty}^{\infty} f(x)dx = \frac{2k}{3} = 1$, $k = 1.5$.

(b). The cumulative distribution

$$F(x) = \int_{-\infty}^{x} f(t)dt = \begin{cases} x\sqrt{x} & \text{0<x<1} \\ 1 & x \geq 1 \\ 0 & x \leq 0 \end{cases}$$

$$P(0.3 < x < 0.6) = F(0.6) - F(0.3) = 0.6\sqrt{0.6} - 0.3\sqrt{0.3} = 0.3004$$

4. The waiting time, in hours, between successive speeders spotted by a radar unit is the continuous random variables with cumulative distribution.

$$F(x) = \begin{cases} 0 & x \leq 0 \\ 1 - e^{-8x} & x > 0 \end{cases}$$

   Find the probability of waiting less than 12 minutes between successive speeders.

   (a) using the cumulative distribution of x.
   (b) using the probability density function of x.

Solution:

$$x = \frac{12}{60} = 0.2$$

(a). $F(x = 0.2) = 1 - e^{-1.6} = 0.7981$.

(b). probability density function is: $f(x) = \frac{dF}{dx} = \begin{cases} 0 & x \leq 0 \\ 8e^{-8x} & x > 0 \end{cases}$.

The probability of waiting less than 12 minutes (0.2 hr) will be

$$\int_{-\infty}^{0.2} f\left(x\right)dx = \int_{0}^{0.2} f\left(x\right)dx = 1 - e^{-8*0.2} = 0.7981.$$